

N 325

$$\delta'(\cos x) = \sum_{k=-\infty}^{\infty} [\delta(x - x_k) / |y'(x_k)|]$$

x_k - розв'язок рівняння $y(x) = 0$

$$\cos x = 0$$

$$x_k = \frac{\pi}{2} + k\pi, \quad k = 0, \pm 1, \dots$$

$$|y'(x_k)| = 1$$

$$\delta'(\cos x) = \sum_{k=-\infty}^{\infty} \delta(x - \frac{\pi}{2} + k\pi)$$

N 328

$$x^m \int \frac{1}{x} = x^{m-1}$$

$$(x^m \int \frac{1}{x}, \varphi) = (\int \frac{1}{x}, x^m \varphi) = \int \frac{x^m}{x} \varphi(x) dx = (x^{m-1}, \varphi(x))$$

N 329 $x^n \delta(x) = 0, \quad n \in \mathbb{N}$

Нехай $\varphi \in \mathcal{D} \Rightarrow$

$$(x^n \delta(x), \varphi(x)) = (\delta(x), x^n \varphi(x)) = \varphi(0) = 0$$

N 332

$$\delta'(x^3 - 7x + 6) = \frac{1}{4} \delta'(x - 1) + \frac{1}{5} \delta'(x - 2) + \frac{1}{20} \delta'(x + 3)$$

$$x^3 - 7x + 6 = 0$$

$$x_k \Rightarrow x_1 = -3$$

$$x_2 = 1$$

$$x_3 = 2$$

$$\delta'(x^3 - 7x + 6) = \frac{1}{4} \delta'(x_2 - 1) + \frac{1}{5} \delta'(x_3 - 2) + \frac{1}{20} \delta'(x_1 + 3)$$

N343

$$\begin{aligned} U(x) &= x^2 \delta'(x) \\ (U(x), \varphi) &= (x^2 \delta', \varphi) = (\delta', x^2 \varphi) = -(\delta, (x^2 \varphi)') = \\ &= -(\delta, \varphi + 2x\varphi') = -(\delta, \varphi) - (\delta, 2x\varphi') = -\varphi(0) - 2x\varphi' \Big|_{x=0} = \\ &= -\varphi(0) = (-\delta, \varphi) \end{aligned}$$

N344 $(U(x)) = x^2 \delta''(x)$

$$\begin{aligned} (U(x), \varphi) &= (x^2 \delta'', \varphi) = (\delta'', x^2 \varphi) = (\delta'', (\varphi + x^2 \varphi'')) = \\ &= (\delta'', 2\varphi' + 2x\varphi') = 2(\delta'', \varphi') + 2(\delta'', x\varphi') = \\ &= 2\varphi'(0) + 2x\varphi'(x) \Big|_{x=0} = 2\varphi'(0) + 2\varphi(x) = \\ &= (2\delta'(x), 2\varphi(x)) \end{aligned}$$